

Many-particle correlations in non-equilibrium Luttinger liquid

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We develop an operator-based approach to the problem of Luttinger liquid conductor in a non-equilibrium stationary state. We show that the coherent-state many-body fermionic density matrix as well as all fermionic correlation functions out of equilibrium are given by one-dimensional functional determinants of the Fredholm type. Thus, the model constitutes a remarkable example of a many-body problem where all the correlation functions can be evaluated exactly. On the basis of the general formalism we investigate four-point correlation functions of the fermions coming out of the Luttinger liquid wire. Obtained correlations in the fermionic distribution functions represent the combined effect of interaction and non-equilibrium conditions.

I. INTRODUCTION

It was understood long ago¹ that conventional description of interacting fermionic systems in the framework of Landau Fermi-liquid theory is not applicable in one dimension (1D) because of infrared divergences. In particular, the second order of perturbation theory yields a singularity in the self-energy at the Fermi surface, $\Re\Sigma \sim \epsilon \ln \epsilon$, implying a vanishing quasi-particle weight. This violates the one-to-one correspondence (which plays the central role in the construction of the Landau theory) between unperturbed electronic states and elementary excitations of the interacting system. The lack of correspondence is a hallmark of the emerging strongly correlated electronic state - Luttinger liquid (LL).

The physics of LL is known to be relevant for many systems available in experiment. The applications of this concept include carbon nanotubes²⁻⁴, semiconducting and metallic nanowires^{5,6}, edge states of the samples in the quantum Hall regime⁷⁻⁹ and spin ladders^{10,11}.

In view of singular infrared behavior, it is hard to access LL by conventional methods of many-body fermion perturbation theory. To overcome this difficulty, a powerful approach—the bosonization—was developed. It is based on the fact that in 1D fermionic creation and annihilation operators have simple representations in terms of bosonic fields describing such observable as charge and spin density. Thus, any 1D fermionic system is equivalent (as long as considered energies are not too high) to some (generally interacting) bosonic system. This Fermi-Bose equivalence was first discovered on the level of correspondence between correlation functions in fermionic and bosonic theories¹²⁻¹⁷. Later on, underlying operator relations were derived by Haldane¹⁸, providing the solid basis for bosonization (for a recent detailed exposition, see¹⁹).

Bosonization has proven to be a very efficient tool for tackling 1D interacting fermions. In some cases it allows to obtain exact solutions of the problems that are highly non-trivial in the fermionic language. The canonical example of this is the Tomonaga-Luttinger model equivalent to free bosons. Even in the case when bosonization does not produce free bosons, it often constitutes a convenient starting point for the development of the theory. Using this approach, effects of backscattering^{20,21}, impurities²², and underlying periodic potential²³ on the LL were explored. Many particular realizations of the LL states were addressed, both theoretically and experimentally^{9,24}.

Nowadays, there is a growing interest in non-equilibrium phenomena in the LL phase. In particular, in a recent experiment²⁵, the tunneling spectroscopy of a biased LL was carried out. A similar approach was implemented to study experimentally carbon nanotubes²⁶ and quantum Hall edges²⁷⁻²⁹. The experimental advances motivated theoretical interest to quantum wires out of equilibrium³⁰⁻³⁹. In a related line of research, non-equilibrium chiral 1D systems have been studied theoretically⁴⁰⁻⁴³, mainly in application to experiments on quantum Hall edge-states interferometry.

In a series of papers by two of the authors and Gefen^{30,31} the theory of non-equilibrium LL was developed. The formalism developed in these works combined the bosonization with the non-equilibrium Keldysh action approach. It was shown that the calculation of single-particle Green function reduces to the evaluation of certain Fredholm determinants, analogous to those encountered in the context of counting statistics⁴⁴ and non-equilibrium orthogonality catastrophe^{45,46}. This allowed to obtain comprehensive results for observables related to the fermionic single-particle Green function, including the tunneling density of states, electron distribution function, and Aharonov-Bohm signal. More recently, the counting statistics of the charge transfer in a non-equilibrium LL was studied in³².

However, a number of fundamental questions have remained open. In particular, it is important to understand what kind of correlations (if any) are induced by the interaction between the electrons coming out of an interacting wire. In particular, are the left- and right-movers that have passed through the wire correlated? We will see below that at equilibrium no such correlations exist. On the contrary, in a non-equilibrium LL electrons experience a specific type of relaxation. We will show that, in course of this relaxation process, non-trivial correlations in the electronic distributions are built. To characterize these correlations is the main task of this work.

Our approach to the problem is complementary to the one of Ref. 31. Using the operator formalism, we determine explicitly the many-body density matrix $\hat{\rho}$ of the LL conductor in a stationary non-equilibrium state. When written in the bosonic coherent-state basis, this non-equilibrium density matrix $\hat{\rho}$ has a form of a Fredholm determinant. The density matrix carries full information about many-body correlations in the non-equilibrium LL state. We find that the calculation of fermionic correlation functions with density matrix $\hat{\rho}$ is greatly simplified by using a “re-fermionization” procedure. In this way, we can evaluate all fermionic correlation functions, with results expressed in terms of Fredholm determinants. Having developed our general formalism, we employ it to study four-point correlation functions of fermionic fields out of equilibrium and explore the correlations in the occupation numbers of the outgoing electrons induced by the interaction.

The structure of the paper is as follows. Section II contains a description of a model of a LL conductor out of equilibrium. In Sec. III we give a short review of bosonization and re-fermionization and introduce concepts needed in the main part of the paper. In Sec. IV we obtain the many-body density matrix of a LL. Further, we show how the bosonization-refermionization allows one to obtain correlation functions of a non-equilibrium LL. We verify that the results for the single-particle Green function agrees with that obtained previously in Ref. 31. In Sec. V we explore the four-point correlation functions of fermions that emerged out of the LL wire. Our result reveals non-trivial correlations in fermionic occupation numbers. Section VI contains a summary of our findings.

II. THE MODEL

Let us specify our model of the non-equilibrium LL conductor. We consider a 1D wire populated by spinless electrons of two chiralities (labeled by $\eta = R, L$) interacting via local density-density interaction. The wire is connected to non-interacting electrodes. To model the later we will assume that the electron-electron interaction is switched off outside the central part of the wire (Fig. 1) so that the Hamiltonian of the problem reads

$$H_{ee} = H_0 + \frac{1}{2} \int dx g(x) (\rho_R^2(x) + \rho_L^2(x)) . \quad (1)$$

Here H_0 represents the kinetic energy part of the Hamiltonian; $\rho_{R(L)}(x)$ is the density of right- (left-) moving electrons at point x . The function $g(x)$ approaches a constant value deep inside the interval $|x| < l/2$ (region II of Fig. 1) and is zero for $|x| > l/2$ (regions I and III). This way of representing the non-interacting leads was used extensively in the literature for the analysis of the transport properties of Luttinger liquids^{47–49}.

The non-equilibrium state of the wire is induced by the injection of the electrons from the leads. Following Refs. 31, we assume that the right- moving electrons coming into the wire from the left lead have a stationary non-equilibrium distribution function $n_R(\epsilon)$, while the left-moving electrons coming from the right lead are characterized by the distribution function $n_L(\epsilon)$. The simplest non-equilibrium state arises when the leads are held at different temperatures T_R , T_L and different chemical potentials μ_R , μ_L so that the distribution functions of the incoming electrons are of the Fermi-Dirac form

$$n_\eta(\epsilon) = \frac{1}{1 + e^{(\epsilon - \mu_\eta)/T_\eta}} . \quad (2)$$

Such a state was named the “partial equilibrium” in Ref.³¹. More complicated distribution functions n_η , e.g. double-step distributions, can be generated if one assumes that the leads are diffusive conductors. We refer the reader to the Ref.³¹ for the comprehensive discussion of the possible experimental realizations of the non-equilibrium Luttinger liquids.

Throughout the paper we assume the absence of the electron backscattering in the wire which can be caused e.g. by impurities. In order to avoid the electronic backscattering also from the boundaries of the interaction region we suppose that the function $g(x)$ is smooth on the length scale of the Fermi wavelength. Withing this limitation, the formalism developed in this work can be applied to the case of arbitrary dependence of the interaction constant on x . However, the most interesting situation arises when boundaries of the interaction region are sharp on the scale of the typical wavelength $l_{T^*} = V_F/T^*$ of the *bosonic* excitations. Here V_F is the Fermi velocity and T^* is some characteristic width of the distribution functions n_η (e.g. $T^* \sim \max(T_R, T_L)$ in the case of the partial equilibrium).

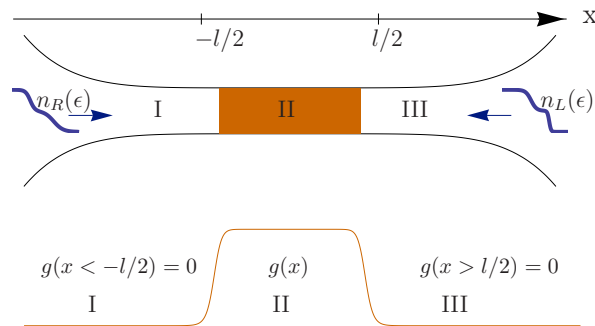


FIG. 1: (Color online). Schematic view a LL conductor driven out of equilibrium by the injection of non-equilibrium electrons (with distribution functions $n_\eta^{\text{in}}(\epsilon)$) from non-interacting leads. The leads are modeled via the assumption that the LL interaction constant $g(x)$ is space dependent and vanishes outside the central part of the wire ($|x| < l/2$, region II).

Under such circumstance, there is significant scattering of the *bosonic excitations* on the boundaries of the interaction region which is the primary source of the effects we are going to discuss in this paper. For this reason we concentrate below on this “sharp boundary” limit and model the x -dependence of the interaction constant by

$$g(x) = g\Theta(l/2 - |x|) . \quad (3)$$

In Ref.³¹ the single particle correlation functions of the model described above were investigated in great detail. While within the interacting part of the wire those correlation functions are highly non-trivial due to the combined effect of interaction and non-equilibrium, outside the interaction region the single particle Green functions are much simpler, reflecting the free dynamics of the electrons. Specifically, the single particle Green functions of the right electrons to the right from the interacting region (region III) has a form of the Green functions of free fermions with some energy distribution $n_R^{\text{out}}(\epsilon)$. In the same way, the left-movers in the region I can be viewed as free particles with energy distribution $n_L^{\text{out}}(\epsilon)$. The correlation functions of incoming electrons, i.e. right-movers in the region I and left-movers in the region III, are not affected by the interaction. The distribution functions n_η^{out} differ from n_η (except for the case of complete thermal equilibrium), signaling the redistribution of the electrons over energies upon the passage through the interacting region II.

Now an important question arises. Is this redistribution (which, crudely speaking, can be referred to as relaxation, although the resulting electronic distribution functions are not of the Fermi-Dirac type) is the only effect of interaction on the outgoing fermions? In particular, are the left- and right-movers coming out of the wire independent? Note that energy relaxation in our device is of a rather special character. Due to the strong constraints imposed by the integrability of the LL model, no energy relaxation can occur in a uniform LL. The relaxation actually takes place at the boundaries of the interacting wire and involves the scattering of the bosonic excitations from one chiral branch to another. Since bosons are crudely speaking electron-hole pairs, we may anticipate that the relaxation should be accompanied by the buildup of correlations in the distribution functions of left- and right-electrons. We show below that this is indeed the case and the density matrix of outgoing fermions shows strong correlations. In particular, the irreducible correlation functions of the form $\langle\langle n_\eta(x, \epsilon) n_{\eta'}(x', \epsilon') \rangle\rangle$ between the occupation numbers of the outgoing fermions are non-zero. These emerging correlations are in the focus of the present work. We stress that the correlations in question are specifically non-equilibrium effect absent under the equilibrium conditions.

We are now ready to present the formalism that we use in this work to explore correlations in a non-equilibrium LL wire. We start with a short review of the standard bosonization approach (in the form of “constructive bosonization”^{18,19}) in application to equilibrium interacting fermions. We make particular emphasis on constructions that will be subsequently employed in the analysis of the non-equilibrium state.

III. BOSONIZATION AND REFORMIONIZATION

Let us consider the 1D fermionic system with the Hamiltonian

$$H = \int dx \psi^\dagger(x) \frac{\hat{p}^2 - k_F^2}{2m} \psi(x) + V_{\text{int}} . \quad (4)$$

Here k_F is the Fermi momentum and V_{int} represents the four-fermion interaction. Within the Luttinger liquid model one linearizes the spectrum of the fermions near the Fermi points and adds unphysical (but irrelevant at energies

smaller than the Fermi energy) states below the bottom of the Fermi sea. This way of treating the interacting fermions is justified for the description of low-energy properties of the system. At high energies effects related to the curvature of the spectrum (not included in the LL model) may become sizeable, see, in particular Ref. 50.

With the bottom of the Fermi sea pushed down to minus infinity, the Hilbert space of the problem is spanned by the fermionic creation and annihilation operators $a_{\eta,k}^+$, $a_{\eta,k}$ labeled by momentum $-\infty < k < +\infty$ (counted from k_F) and the chirality $\eta = R, L$ (in formulas, we will also use the notation $\eta = 1$ for right-movers and $\eta = -1$ for left-movers). It is convenient to make the fermionic momenta discrete, assuming that the system is placed on a ring of large circumference L . The thermodynamic limit $L \rightarrow \infty$ is taken at the end. The physical fermionic field is decomposed into the sum of left- and right-moving parts according to ($\Lambda \rightarrow \infty$ stands for the high energy cutoff)

$$\psi(x) \sim \psi_R(x)e^{ik_F x} + \psi_L(x)e^{-ik_F x}, \quad (5)$$

$$\psi_\eta(x) = \frac{1}{\sqrt{L}} \sum_k a_{k,\eta} e^{ikx - |k|/\Lambda}. \quad (6)$$

The Hamiltonian of the LL model acquires the form

$$H = V_F \sum_{k,\eta} \eta k \left(a_{\eta,k}^+ a_{\eta,k} - n_\eta^0(k) \right) + \frac{1}{2} \int dx g(x) (\rho_L(x) + \rho_R(x))^2. \quad (7)$$

Here $n_\eta^0(k) = \Theta(-\eta k)$ are occupation numbers of the left- and right-movers in the free-fermion ground state; the interaction term was taken in the form of local density-density interaction with space-dependent interaction constant as discussed in the previous section.

The central role in the Luttinger-liquid model is played by the Fourier components of fermionic densities

$$\rho_{\eta,q} = \sum_k a_{\eta,k+q}^+ a_{\eta,k} \quad (8)$$

which can be used to construct the set of bosonic creation and annihilation operators according to ($\eta q > 0$):

$$b_{\eta,q}^+ = \sqrt{\frac{2\pi}{L|q|}} \rho_{\eta,q}, \quad b_{\eta,q} = \sqrt{\frac{2\pi}{L|q|}} \rho_{\eta,-q}, \quad (9)$$

$$[b_{\eta,q}, b_{\eta',q'}^+] = \delta_{\eta\eta'} \delta_{qq'}. \quad (10)$$

Let us denote by $|N_R, N_L\rangle$ the state of the system which is a filled Fermi sea with $N_{R(L)}$ extra particles in the right (left) branch, i.e the state characterized by the distribution functions $n_{\eta,k}^{N_\eta} = \Theta(-\eta k + 2\pi N_\eta/L)$. All these states are annihilated by $b_{\eta,q}$ and are vacuum states from the point of view of the bosons. Any other state of the fermions can be generated by the action of the bosonic raising operators onto $|N_R, N_L\rangle$. Thus, the operators $b_{\eta,q}, b_{\eta,q}^+$ together with the particle number operators N_η and the Klein factors F_η, F_η^+ changing the total number of fermions of corresponding chirality form the complete operator set. In particular, the free-fermion Hamiltonian H_0 can be reexpressed in terms of bosons as

$$H_0 = V_F \sum_{\eta,q} \Theta(\eta q) |q| b_{\eta,q}^+ b_{\eta,q} + \frac{\pi V_F}{L} \sum_\eta N_\eta (N_\eta + 1), \quad (11)$$

while the fermionic field operators are given by the famous bosonization identity

$$\psi_\eta^+(x) = \sqrt{\frac{\Lambda}{4\pi}} e^{-i\varphi_\eta(x)} F_\eta^+. \quad (12)$$

The phase $\varphi_\eta(x)$ is related to the corresponding density $\rho_\eta(x)$ via

$$\rho_\eta(x) = \frac{\eta}{2\pi} \partial_x \varphi_\eta(x). \quad (13)$$

Explicitly, in terms of bosonic creation and annihilation operators the density and phase fields read as follows,

$$\varphi_\eta(x) = i \sum_q \Theta(\eta q) \sqrt{\frac{2\pi}{L|q|}} (e^{-iqx} b_{q,\eta}^+ - e^{iqx} b_{q,\eta}) + \frac{2\pi\eta}{L} N_\eta x, \quad (14)$$

$$\rho_\eta(x) = \sum_{\eta,q} \Theta(\eta q) \sqrt{\frac{|q|}{2\pi L}} (e^{-iqx} b_{\eta,q}^+ + e^{iqx} b_{\eta,q}) + \frac{1}{L} N_\eta. \quad (15)$$

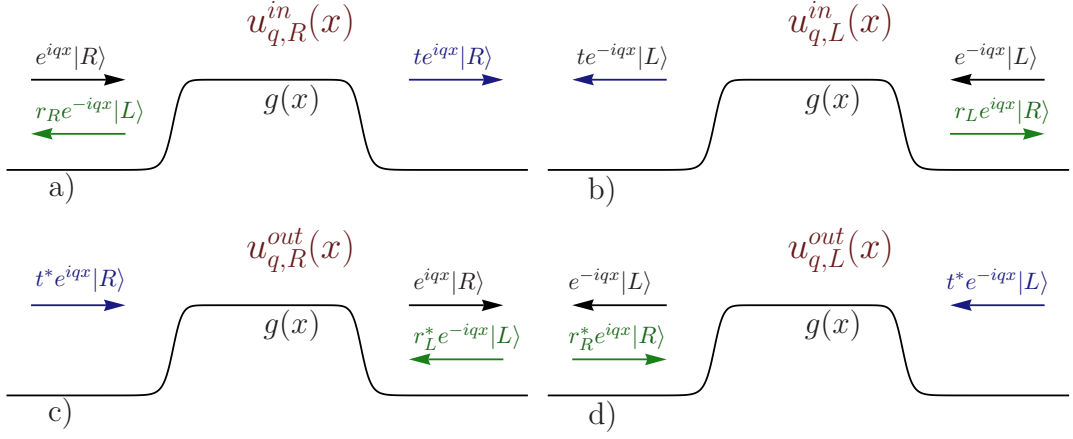


FIG. 2: (Color online). a), b) Bosonic wave functions of the in- scattering states outside the interacting part of the wire. c), d) Bosonic wave functions of the out- scattering states.

In Eqs. (11, 14, 15) the last terms involving the total number of fermions are usually referred to as the "zero mode" contributions to the energy, bosonic phase and the density. We keep them in the equations above in order to maintain the completeness of our short presentation of the bosonization technique. On the other hand, these contributions do not play any significant role in the non-equilibrium effects we are going to discuss below. For this reason, we omit the zero-mode terms altogether in the subsequent formulas.

The power of the bosonization approach rests upon the fact that the interacting fermionic Hamiltonian (7) can be reexpressed as a *quadratic* function of bosonic operators. Thus, H can be diagonalized by linear transformation of bosons. Below we will need explicit expressions for the bosons diagonalizing H . To derive them, it is convenient to start with the equations of motion for the density operators $\rho_\eta(x, t)$,

$$\frac{\partial}{\partial t} \begin{pmatrix} \rho_R(x, t) \\ \rho_L(x, t) \end{pmatrix} + \frac{\partial}{\partial x} \left[\begin{pmatrix} V_F + \frac{g(x)}{2\pi} & \frac{g(x)}{2\pi} \\ -\frac{g(x)}{2\pi} & -V_F - \frac{g(x)}{2\pi} \end{pmatrix} \begin{pmatrix} \rho_R(x, t) \\ \rho_L(x, t) \end{pmatrix} \right] = 0. \quad (16)$$

We decompose the solution of the equation above into the normal modes according to

$$\rho_\eta(x, t) = \sum_{q, \eta'} \Theta(\eta' q) \sqrt{\frac{|q|}{2\pi L}} \left(u_{\eta', |q|}^*(\eta, x) \tilde{b}_{\eta', q}^+ e^{i|q|V_F t} + u_{\eta', |q|}(\eta, x) \tilde{b}_{\eta', q} e^{-i|q|V_F t} \right). \quad (17)$$

Here \tilde{b} and \tilde{b}^+ are the new bosonic operators, and $u_{\eta', |q|}(\eta, q)$ are the coefficient functions. It is convenient to organize the latter into two-component columns $u_{\eta, q}(x) = (u_{\eta, q}(x, R), u_{\eta, q}(x, L))^T$ satisfying the equation

$$-iV_F q u_{\eta, q}(x) + \frac{\partial}{\partial x} [\tau_z h(x) u_{\eta, q}(x)] = 0, \quad (18)$$

where τ_z is the third Pauli matrix and

$$h(x) = \begin{pmatrix} V_F + \frac{g(x)}{2\pi} & \frac{g(x)}{2\pi} \\ \frac{g(x)}{2\pi} & V_F + \frac{g(x)}{2\pi} \end{pmatrix}. \quad (19)$$

Note that in order to construct the density fields we need only the solutions of Eq. (18) with $q > 0$. Thus, in the discussion of the properties of $u_{\eta, q}(x)$ we will assume that $q > 0$. On the other hand, for $q < 0$ one can define $u_{\eta, q}(x)$ via the complex conjugate

$$u_{\eta, q < 0}(x) \equiv u_{\eta, -q}^*(x). \quad (20)$$

This choice is consistent with Eq. (18).

Imposing different boundary conditions, one can construct two sets of solutions of the Eq. (18). We denote them by ($q > 0$): $u_{q, \eta}^{in}$ and $u_{q, \eta}^{out}$. Far from the interacting region the solution $u_{q, R(L)}^{in}$ consists of a right (left) incoming wave and the waves scattered of the interacting region. These are the *in*-states in the standard terminology of the

scattering theory. On the other hand $u_{q,R}^{out}$ and $u_{q,L}^{out}$ represent the *out*- scattering states. They contain the outgoing wave and the waves incident on the barrier. The asymptotic behavior of $u_{\eta,q}^{in(out)}$ is summarized on Fig. 2. We have introduced the bosonic q -dependent scattering coefficients of the interaction region. Due to the time reversal symmetry the transmission amplitude t_q for the right wave $u_{q,R}^{in}$ coincides with that of the left wave, whereas reflection amplitudes for the right and left waves are related via $r_R^*/r_L = -t^*/t$. If we assume the boundaries of the interaction region to be sharp on the scale of the relevant bosonic wavelength and adopt the model of the rectangular-shaped interaction constant (3) we find:

$$t_q = \frac{\mathcal{T}^2 e^{-i(1-K)ql}}{1 - \mathcal{R}^2 e^{2iKql}}, \quad (21)$$

$$r_{R,q} = r_{L,q} = -2i\mathcal{R} \frac{e^{-i(1-K)ql}}{1 - \mathcal{R}^2 e^{2iKql}} \sin Kql. \quad (22)$$

Here K is the LL parameter of the interaction region, while $\mathcal{R} = (1-K)/(1+K)$ and $\mathcal{T} = \sqrt{1-\mathcal{R}^2} = 2\sqrt{K}/(K+1)$ are reflection and transmission amplitudes of a single interaction boundary.

Clearly, outgoing solutions $u_{q,\eta}^{out}$ are not independent of $u_{q,\eta}^{in}$. The two sets of solutions are related by the elements of the bosonic scattering matrix of the interaction region r_q and t_q :

$$u_{R,q}^{out} = t_q^* u_{R,q}^{in} + r_{L,q}^* u_{L,q}^{in}, \quad (23)$$

$$u_{L,q}^{out} = r_{R,q}^* u_{R,q}^{in} + t_q^* u_{L,q}^{in}. \quad (24)$$

Choosing $u_{\eta,q}^{in}(x)$ as the coefficient functions in Eq. (17), we obtain the decomposition of the density in terms of the in- bosons b^{in} and b^{+in} ,

$$\rho_\eta(x) = \sum_{q',\eta'} \Theta(\eta'q') \sqrt{\frac{|q'|}{2\pi L}} \left(u_{\eta',|q'|}^{in*}(\eta, x) b_{\eta',q'}^{+in} + u_{\eta',|q'|}^{in}(\eta, x) b_{\eta',q'}^{in} \right). \quad (25)$$

Comparing (25) to (15), one can read off the relation of the initial and in-bosons:

$$b_{\eta,q}^{+in} = \sum_{\eta',q'} \Theta(\eta'q') \left[u_{|q|,\eta}^{in}(q', \eta') b_{\eta',q'}^+ - u_{|q|,\eta}^{in}(-q', \eta') b_{\eta',q'} \right]. \quad (26)$$

Operators $b_{\eta,q}^{in}$ are defined via hermitian conjugation and $u_{q,\eta}^{in}(q')$ are the Fourier components of $u_{q,\eta}^{in}(x)$:

$$u_{q,\eta}^{in}(q', \eta') = \frac{1}{L} \sqrt{\frac{|q'|}{|q|}} \int dx e^{-iq'x} u_{q,\eta}^{in}(x, \eta'). \quad (27)$$

By construction, operators $b_{q,\eta}^{in}$ and $b_{q,\eta}^{+in}$ diagonalize the Hamiltonian H :

$$H = V_F \sum_{\eta,q} \Theta(\eta q) |q| b_{\eta,q}^{in+} b_{\eta,q}^{in}. \quad (28)$$

The connection (13) of the phase fields φ_η to the densities can now be used to derive the relation between φ_η and the incoming bosons,

$$\varphi_\eta(x) = \frac{i}{V_F} \sum_{q',\eta'} \Theta(\eta'q') \sqrt{\frac{2\pi}{L|q'|}} \left(v_{\eta',|q'|}^{in*}(\eta, x) b_{\eta',q'}^{+in} - v_{\eta',|q'|}^{in}(\eta, x) b_{\eta',q'}^{in} \right). \quad (29)$$

Here we have introduced the two-component columns $v_{\eta,q}^{in}(x)$ given by (the matrix $h(x)$ was defined in (19))

$$v_{\eta',q'}^{in}(x) = h(x) u_{\eta',q'}^{in}(x). \quad (30)$$

Since $\psi_\eta^+(x) \sim \exp[-i\varphi_\eta(x)]$, equation (29) connects the fermionic field to the in-bosons.

For the purposes of this work, it will be convenient to perform *refermionization* of the in- bosons $b_{\eta,q}^{in}$ by introducing new fermionic fields $\psi_{\eta}^{in}(x)$ and their Fourier components $a_{\eta,k}^{in}$ according to

$$\psi_{\eta}^{+in}(x) = \frac{1}{\sqrt{L}} \sum_k a_{\eta,k}^{+in} e^{-ikx} = \frac{1}{\sqrt{L\epsilon}} e^{-i\varphi^{in}(x)} F_{\eta}^{+}, \quad (31)$$

$$\varphi_{\eta}^{in}(x) = i \sum_q \Theta(\eta q) \sqrt{\frac{2\pi}{L|q|}} (e^{-iqx} b_{q,\eta}^{+in} - e^{iqx} b_{q,\eta}^{in}). \quad (32)$$

The inverse relation reads

$$b_{\eta,q}^{+in} = \sqrt{\frac{2\pi}{L|q|}} \sum_k a_{\eta,k+q}^{+in} a_{\eta,k}^{in}, \quad b_{\eta,q}^{in} = \sqrt{\frac{2\pi}{L|q|}} \sum_k a_{\eta,k-q}^{+in} a_{\eta,k}^{in}. \quad (33)$$

Since the in- bosons solve the LL Hamiltonian, the in- fermions $a_{\eta,k}^{in}$ are just free fermions with the time evolution

$$a_{\eta,k}^{in}(t) = a_{\eta,k}^{in} e^{-i\eta k v_F t}. \quad (34)$$

Comparing Eqs. (29) and (32) and taking into account the boundary conditions imposed on $u_{\eta,q}^{in}$, we see that for $\eta x \rightarrow -\infty$ (e.g., for right fermions at $x \rightarrow -\infty$) the physical fermionic field $\psi_{\eta}(x)$ is identical with $\psi_{\eta}^i(x)$,

$$\psi_{\eta}(x) = \psi_{\eta}^{in}(x), \quad \eta x \rightarrow -\infty. \quad (35)$$

So far, we were focussing on the in- solutions $u_{\eta,q}^{in}$. The out- solutions $u_{\eta,q}^{out}$ allow us to define the out- bosons $b_{\eta,q}^{out}$ and fermions $a_{\eta,k}^{out}$ by equations analogous to (26), (31), and (32) up to a replacement $in \rightarrow out$ everywhere. These out- fermions and bosons also solve the LL Hamiltonian. Just as the solutions $u_{\eta,q}^{in}$ and $u_{\eta,q}^{out}$, the bosons $b_{\eta,q}^{in}$ and $b_{\eta,q}^{out}$ are related by the bosonic transmission and reflection coefficients,

$$b_{R,q}^{+out} = t_q^* b_{R,q}^{+in} + r_{L,q}^* b_{L,-q}^{+in}, \quad (36)$$

$$b_{L,q}^{+out} = r_{R,q}^* b_{R,-q}^{+in} + t_q^* b_{L,q}^{+in}. \quad (37)$$

The out- fermions represent the physical fermions after the crossing of the interaction region in the sense that (cf. Eq.(35)):

$$\psi_{\eta}(x) = \psi_{\eta}^{out}(x), \quad \eta x \rightarrow +\infty. \quad (38)$$

The refermionization procedure described above turns out to be the crucial step in the solution of the non-equilibrium LL model. The in- fermions introduced in (31) will provide us with the starting point for the construction of the non-equilibrium density matrix of a LL wire. The out- fermions will be useful in our discussion of the correlations among the electrons leaving the wire.

IV. NON-EQUILIBRIUM LUTTINGER LIQUID

A. Density matrix

In the previous section we have summarized the bosonization representation of the LL Hamiltonian. Let us now turn to the analysis of the non-equilibrium LL in the setup shown on Fig. 1. First, we need to understand what is the (many-body) density matrix $\hat{\rho}$ describing the situation of Fig. 1. This density matrix should satisfy two conditions:

- a) it should be stationary with respect to the LL Hamiltonian (7);
- b) any correlation function of the fermionic fields $\langle \psi_{\eta_1}(x_1, t_1) \dots \psi_{\eta_i}^{+}(x_i, t_i) \dots \rangle$ evaluated with the density matrix $\hat{\rho}$ should be identical to the same correlation function of free fermions with the density matrix

$$\hat{\rho}_0 = \frac{1}{Z} \exp \left[- \sum_{\eta,k} \epsilon_{\eta}(k) \left(a_{\eta,k}^{+} a_{\eta,k} - n_{\eta}^0(k) \right) \right] \quad (39)$$

as soon as all coordinates of the fermionic fields satisfy $\eta_i x_i < -l/2$. In Eq. (39) Z stands for the normalization factor and the parameters $\epsilon_\eta(k)$ are determined by the distribution functions of the fermions coming into the wire via

$$n_\eta(k) = \frac{1}{1 + e^{\epsilon_\eta(k)}}. \quad (40)$$

Both conditions a) and b) are fulfilled by the density matrix

$$\hat{\rho} = \frac{1}{Z} \exp \left[- \sum_{\eta, k} \epsilon_\eta(k) \left(a_{\eta, k}^{+in} a_{\eta, k}^{in} - n_\eta^0(k) \right) \right]. \quad (41)$$

The condition a) is satisfied due to the equation of motion (34) for the incoming fermions. The condition b) is satisfied due to the fact that the fermionic fields $\psi_{\eta_i}(x_i)$ are just identical to $\psi_{\eta_i}^{in}(x_i)$ as soon as $\eta_i x_i \rightarrow -\infty$.

In equilibrium, $\epsilon_\eta(k) = \eta k V_F / T$, the density matrix in terms of bosons is simply given by

$$\hat{\rho}_{eq} \sim \exp \left[- \frac{V_F}{T} \sum_{q, \eta} \Theta(\eta q) |q| b_{\eta, q}^{+in} b_{\eta, q}^{in} \right]. \quad (42)$$

Together with Eq. (29) this implies that the evaluation of the fermionic correlation functions in equilibrium is reduced to the evaluation of the averages of the type ($g_\eta(q)$ and $g_\eta^*(q)$ are c-valued functions)

$$Z[g_\eta^*, g_\eta] \equiv \left\langle \exp \left[\sum_{\eta, q} \Theta(\eta q) g_\eta(q) b_{\eta, q}^{in+} \right] \exp \left[- \sum_{\eta, q} \Theta(\eta q) g_\eta^*(q) b_{\eta, q}^{in} \right] \right\rangle \quad (43)$$

with the gaussian weight (42), which is a straightforward task.

Translating the general non-equilibrium density matrix (41) into the bosonic language is a much more non-trivial task. As shown in Appendix A, it turns out to be possible to evaluate the matrix elements of $\hat{\rho}$ in the basis of bosonic coherent state $|N, \beta\rangle$ (eigenstates of the bosonic annihilation operators $b_{\eta, q}^{in}$) in the form of one-dimensional Fredholm determinants. Here we state only the final result of this analysis, referring the reader to Appendix A for the precise definitions and details of the calculation:

$$\langle \beta_R^*, \beta_L^* | N_R, N_L | \hat{\rho} | N_R, N_L, \beta_R, \beta_L \rangle = D_R(\beta_R^*, \beta_R) D_L(\beta_L^*, \beta_L), \quad (44)$$

$$D_\eta = \det \left[1 - e^{\Phi_\eta(x)} n_\eta^{N_\eta}(k) e^{-\Phi_\eta(x)} (1 - n_\eta(k)) - e^{-\Phi_\eta^+(x)} (1 - n_\eta^{N_\eta}(k)) e^{\Phi_\eta^+(x)} n_\eta(k) \right]. \quad (45)$$

In the determinants (45) the distribution functions $n_\eta(k)$ and $n_\eta^{N_\eta}(k)$ are considered as operators diagonal in the momentum space. On the contrary, operators $\Phi_\eta(x)$ are diagonal with respect to the conjugate variable x and given by

$$\Phi_\eta(k_1 - k_2) = \Theta(\eta(k_1 - k_2)) \sqrt{\frac{2\pi}{L|k_1 - k_2|}} \beta_{\eta, k_1 - k_2}. \quad (46)$$

The density matrix (44), (45), along with the Hamiltonian (28) and the expression for the fermionic operators (29) contains the whole information about the problem in the language of non-interacting bosons. The natural next step is to calculate n -point fermionic correlation functions (that determine various physical observables). This will be done in Sec. IV B

B. Electronic correlation function

Our goal now is to evaluate many-point fermionic Green functions that have the form (43), with the bosonic density matrix given by Eqs. (44), (45). While this can be done by a direct calculation in the bosonic language, such a way turns out to be quite tedious. A shorter way is to perform a refermionization of the bosons b^{in} . Indeed, according to Eq. (33), b^{in+} , b^{in} are quadratic functions of the in-fermions a^{in+} , a^{in} . Further, the density matrix is quadratic in terms of the in-fermions as well, see Eq. (41). Thus, the average (43) can be expressed as trace of a certain operator,

which is an *exponential of a quadratic form* with respect to fermions a^{in} . The evaluation of the trace leads to (see Appendix B for details):

$$Z[g^*, g] = \Delta_R [\delta_R(x)] \Delta_L [\delta_L(x)] , \quad (47)$$

$$\Delta_\eta [\delta_\eta(x)] = \det \left[\left(1 - n_\eta^0(k) + e^{-i\delta_\eta(x)} n_\eta^0(k) \right)^{-1} \left(1 - n_\eta(k) + e^{-i\delta_\eta(x)} n_\eta(k) \right) \right] , \quad (48)$$

$$\delta_\eta(x) = i \sum_q \sqrt{\frac{2\pi}{L|q|}} \Theta(\eta q) (g_{\eta,q} e^{iqx} - g_{\eta,q}^* e^{-iqx}) . \quad (49)$$

In Eq. (48) we can recognize a one-dimensional functional determinant of the type discovered in³¹ in the context of a single-particle Green function. The first factor in square brackets of Eq. (48) involves the zero-temperature distribution function $n_\eta^0(k)$ and serves as a regularization. It ensures that at zero temperature $Z[g^*, g]$, which is an average of a bosonic normal-ordered expression, is identically equal to unity.

Having derived the general result, we turn to evaluation of many-point fermionic correlation functions. Let us consider the average of a product of n operators $\psi_{\eta_i}(x_i, t_i)$, $i = 1 \dots n$ and n operators $\psi_{\eta_i}^+(x_i, t_i)$, $i = n+1 \dots 2n$,

$$M_{\eta_1 \dots \eta_{2n}}(x_1, t_1, \dots, x_{2n}, t_{2n}) = \left\langle \psi_{\eta_1}(x_1, t_1) \dots \psi_{\eta_n}(x_n, t_n) \psi_{\eta_{n+1}}^+(x_{n+1}, t_{n+1}) \dots \psi_{\eta_{2n}}^+(x_{2n}, t_{2n}) \right\rangle . \quad (50)$$

Representing the fermionic fields by bosonic exponents according to (12, 29) and bringing the product of the exponents into the normal ordered form we get

$$M_{\eta_1 \dots \eta_{2n}}(x_1, t_1, \dots, x_{2n}, t_{2n}) = M_{\eta_1 \dots \eta_{2n}}^0(x_1, t_1, \dots, x_{2n}, t_{2n}) Z[g^*, g] , \quad (51)$$

$$g_\eta(q) = -\frac{1}{V_F} \sqrt{\frac{2\pi}{L|q|}} \sum_{i=1}^n \zeta_i v_{\eta,|q|}^{in*}(\eta_i, x_i) e^{i\eta q V_F t_i} , \quad g_\eta^*(q) = -\frac{1}{V_F} \sqrt{\frac{2\pi}{L|q|}} \sum_{i=1}^n \zeta_i v_{\eta,|q|}^{in}(\eta_i, x_i) e^{-i\eta q V_F t_i} . \quad (52)$$

Here $\zeta_i = 1$ for $i = 1, \dots, n$, while $\zeta_i = -1$ for $i = n+1, \dots, 2n$. In the expression (51) the first factor M^0 arises due to normal ordering of the bosons and is nothing but the corresponding correlation function of ψ -operators in zero-temperatures LL. This factor depends in particular on the ordering of ψ -operators in (50). On the other hand, the second factor in (51) accumulates the effect of the distribution functions of the incoming electrons $n_\eta(\epsilon)$. This factor equals unity at zero temperature and is the same for all the fermionic correlators which differ only by the ordering of the Fermi fields. Explicit expression for the factor $Z[g_\eta^*, g_\eta]$ is given by the Fredholm determinants (48) with the phases $\delta_\eta(x)$ expressed in terms of the functions $v_{\eta,q}(x, \eta')$ via [cf. Eqs. (49,52)]

$$\nabla_x \delta_\eta(x) = \frac{2\pi\eta}{V_F} \int \frac{dq}{2\pi} \sum_{i=1}^n \zeta_i v_{\eta,q}^{in}(\eta_i, x_i) e^{-iq V_F t_i - i\eta q x} . \quad (53)$$

Equations (51), (47) and (53) together with (18, 30) provide the full solution of the non-equilibrium LL problem. It is straightforward to check that the two-point (single-particle) correlation functions found previously in Ref.³¹ are correctly reproduced.

Expression for the phases $\delta_\eta(x)$ acquires particularly simple form in the case when all the fermionic fields in (50) are taken outside the interacting part of the wire, i.e. $|x_i| > l/2$ for all $i = 1, \dots, 2n$. Taking into account the boundary conditions imposed on the functions $u_{\eta,q}^{in}$ we get

$$\nabla_x \delta_\eta(x) = 2\pi\eta \int \frac{dq}{2\pi} \sum_i \zeta_i e^{iq(\eta_i u_i - \eta x)} [\Theta(\eta_i x_i) (t_q \delta_{\eta, \eta_i} + r_{\eta,q} \delta_{\eta, -\eta_i}) + \Theta(-\eta_i x_i) \delta_{\eta \eta_i}] . \quad (54)$$

Here we have introduced the light-cone coordinates $u_i = x_i - \eta_i V_F t_i$. According to (54), in the case of the electronic correlations at the input of the central part of the wire (i.e. for all $\eta_i x_i < -l/2$) the phase $\delta_\eta(x)$ are completely independent from the properties of the interaction region. This ensures the coincidence of the correlation functions with that of the free fermions with the density matrix (39). On the other hand, for the correlations of the fermions at the output of the interacting region (i.e. $\eta_i x_i > l/2$) the phases $\delta_\eta(x)$ are determined by the bosonic transmission and reflection amplitudes.

V. CORRELATIONS IN THE OUTGOING FERMIONS

A. Single-particle Green functions

We are now in a position to turn to the correlations between the electrons going out of the interacting part of wire. The simplest quantities characterizing this correlations are the four-point correlation functions of the type (50) where the right fermions are “measured” to the right of the interaction region, while the left electrons are “measured” to the left, i.e. we consider the behavior of Eq. (50) for $\eta_i x_i > l/2$. Intuitively, the electrons which have left the interaction region are just free electrons. We can make this statement mathematically precise by recalling Eq. (38) showing that for $\eta_i x_i > l/2$ physical electronic fields coincide with the fields of out- fermions, having just free *dynamics*

$$\psi_\eta(x, t) = \psi_\eta^{out}(x, t) = \psi_\eta^{out}(x - \eta V_F t), \quad \eta x > l/2. \quad (55)$$

Thus, the question arises, if any of the correlation functions (50) are non-trivial. However, apart from the dynamics, there is another important ingredient of the correlation functions. This is the density matrix.

At equilibrium, the density matrix in terms of the in-bosons is given by (42). Using the interrelation (37) of in- and out- bosons and refermionizing the out-bosons, one readily finds

$$\hat{\rho}_{eq} = \frac{1}{Z} \exp \left[-\frac{1}{T} \sum_{\eta, k} k V_F \left(a_{\eta, k}^{+out} a_{\eta, k}^{out} - n_\eta^0(k) \right) \right]. \quad (56)$$

Thus, at equilibrium the out-electrons have both free dynamics and trivial density matrix. In this situation none of the correlation functions (50) with $\eta_i x_i > l/2$ bears any trace of the interaction in the central part of the wire. In particular, the prefactor M^0 entering Eq. (51) for the correlation functions at the output of the wire is just the free fermion zero temperature correlation function.

The situation changes drastically in a non-equilibrium system. The density matrix $\hat{\rho}$ becomes now a complicated function of the out- fermions, incorporating non-trivial many-particle correlations. It is important to distinguish between the correlation functions which are determined solely by the dynamics and the correlations also governed by the electronic distribution. The retarded and the Keldysh single particle Green functions are the simplest examples of the former and the latter. For the non-interacting electrons $G_\eta^{R0}(x, \tau) \sim \delta(x - \eta V_F \tau)$. Thus, for the retarded function we immediately get

$$G_\eta^R(x, \tau) = G_\eta^{R0}(x, \tau) \Delta_R[\delta_R \equiv 0] \Delta_L[\delta_L \equiv 0] = G_\eta^{R0}(x, \tau). \quad (57)$$

On the other hand, the Keldysh component of the Green function (say for right electrons) $G_R^K(x_R, \tau_R/2; x_R, -\tau_R/2) = -i[\psi_R(x_R, \tau_R/2), \psi_R^\dagger(x_R, -\tau_R/2)] \equiv G^K(\tau_R)$ receives non-trivial contribution from the combined effect of interaction and non-equilibrium,

$$G_R^K(x_R, \tau_R/2, x_R, -\tau_R/2) = G_R^{K0}(0, \tau_R) \Delta_R[\delta_R] \Delta_L[\delta_L], \quad (58)$$

where the phases δ_η are given by

$$\nabla_x \delta_\eta(x) = -4\pi i \eta \int \frac{dq}{2\pi} e^{iq(x_R - \eta x)} \sin \frac{q V_F \tau_R}{2} (t_q \delta_{\eta, R} + r_{L, q} \delta_{\eta, L}). \quad (59)$$

Our choice for the arguments of the fermionic fields in the definition of the Keldysh Green function is somewhat over complicated, since G^K is actually independent of x_R (as long as $x_R > l/2$). However we keep x_R explicit for the convenience of the forthcoming discussion of the four-point correlations.

Exploiting the explicit expression (22) for the transmission and reflection amplitudes in the “sharp boundary” model we get the phases δ_η in the characteristic form of sequence of rectangular pulses of length $v_F \tau_R$ ³¹. The amplitudes of the pulses are shown on Fig. 3. Their positions are given by

$$x_{Rm} = x_R - l + (2m + 1)Kl, \quad m = 0, 1, \dots \quad (60)$$

for $\delta_R(x)$ and

$$x_{Lm} = -x_R + l - 2mKl, \quad m = 0, 1, \dots \quad (61)$$

for the left phase. Analytically

$$\delta_\eta(x) = 2\pi \sum_{m=0}^{\infty} \alpha_{\eta, m} w_{\tau_R}(x - x_{\eta, m}). \quad (62)$$

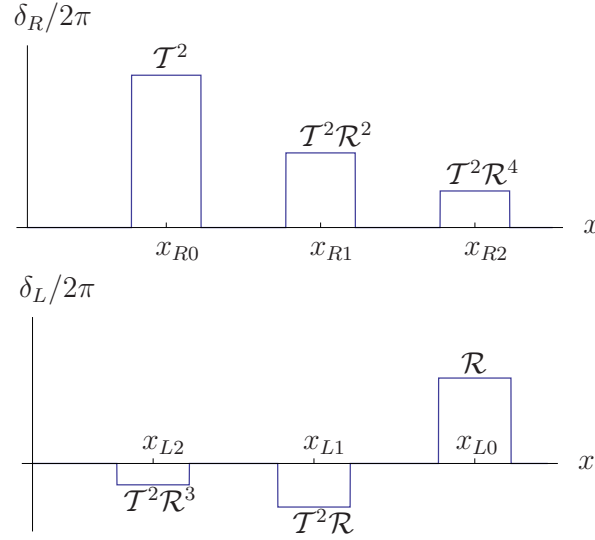


FIG. 3: The phases $\delta_R(x)$ and $\delta_L(x)$ governing the Keldysh Green function of the right-movers. The coordinates $x_{\eta,0}, x_{\eta,1} \dots$ are given by Eqs. (60,61).

Here the coefficients

$$\alpha_{R,m} = \mathcal{T}^2 \mathcal{R}^{2m}, \quad \alpha_{L,m} = \begin{cases} \mathcal{R}, & m = 0 \\ -\mathcal{T}^2 \mathcal{R}^{2m-1}, & m = 1, 2, \dots \end{cases}, \quad (63)$$

and we have introduced

$$w_\tau(x) = \Theta\left(\frac{|\tau|}{2} - |x|\right) \text{sign } \tau. \quad (64)$$

Equation (58) can be interpreted in terms of the fermionic occupation numbers at the output of the wire,

$$1 - 2n_\eta^{\text{out}}(\epsilon) = iV_F \int d\tau_R e^{i\epsilon\tau_R} G_\eta^K(\tau_R). \quad (65)$$

Since the $n_\eta^{\text{out}}(\epsilon)$ are different from the distribution functions of the incoming electrons $n_\eta(\epsilon)$ (except for the equilibrium), electrons experience relaxation upon crossing the interaction region³¹.

B. Two-particle Green functions

Let us now turn to a deeper characterization of the density matrix for the outgoing electrons that is provided by the four-point correlation functions. Particularly intriguing are interaction-induced correlations between the left- and right-movers. To reveal them we consider⁵¹

$$M_{RL}(x_R, \tau_R; x_L, \tau_L) = \langle [\psi_R(x_R, \frac{\tau_R}{2}), \psi_R^\dagger(x_R, -\frac{\tau_R}{2})] [\psi_L(x_L, \frac{\tau_L}{2}), \psi_L^\dagger(x_L, -\frac{\tau_L}{2})] \rangle + G_R^K(\tau_R) G_L^K(\tau_L). \quad (66)$$

were $x_R > l/2$ and $x_L < -l/2$; the square brackets $[\cdot, \cdot]$ stand for the commutator. Note that (66) is the only non-trivial irreducible correlation function of two right and two left fields. Replacement of one of the commutators in (66) by anticommutator would immediately lead to the decoupling of left and right fermions under the average. It is easy to see that the Fourier transform of (66) has the meaning of the irreducible correlator of two fermionic distribution functions,

$$\langle \langle n_R(x_R, \epsilon_R) n_L(x_L, \epsilon_L) \rangle \rangle = \frac{V_F^2}{4} \int d\tau_R d\tau_L M(x_R, \tau_R; x_L, \tau_L) e^{i\epsilon_R \tau_R + i\epsilon_L \tau_L}. \quad (67)$$

Each of the phases $\delta_\eta(x)$ corresponding to (66) is a sum as of a pulse sequence encountered previously in the discussion of the Keldysh function G_R^K and an analogous contribution originating from the pair of left operators in (66). This last contribution consists of pulses of the length $V_F \tau_L$ located at

$$x'_{Rm} = -x_L - l + 2mKl, \quad m = 0, 1, \dots \quad (68)$$

for $\delta_R(x)$ and

$$x'_{Lm} = x_L + l - (2m + 1)Kl, \quad m = 0, 1, \dots \quad (69)$$

for the left phase.

The dependence of (67) on the coordinates x_η and the “optical length” of the wire Kl can be easily understood. We note that the scale l_{T^*}/V_F for the times τ_η supporting non-zero correlation functions is set by the inverse characteristic width of the energy distributions of the incoming electrons. Under the assumption that the length of the interacting wire is large and Kl exceeds l_{T^*} we can neglect the mutual effect of the non-overlapping pulses in $\delta_{R(L)}$ and represent the corresponding determinants $\Delta_\eta[\delta_\eta(x)]$ by a product of determinants for individual pulses. One immediately concludes that occupation numbers of left- and right- movers are uncorrelated unless the pulses coming from right and left operators overlap. The overlap happens if $X_{RL} = x_R + x_L$ is close to odd multiple of Kl . We can now recast (67) into the form

$$\langle\langle n_R(x_R, \epsilon_R) n_L(x_L, \epsilon_L) \rangle\rangle = \sum_{m \in \text{odd}} f_m^{RL}(\epsilon_R, \epsilon_L, X_{RL} - mKl). \quad (70)$$

The functions $f_m^{RL}(\epsilon_R, \epsilon_L, y)$ are independent of l and decay on the distance l_{T^*} . They are given by

$$f_m^{RL}(\epsilon_R, \epsilon_L, y) = -\frac{V_F^2}{4} \int d\tau_L d\tau_R G_R^K(\tau_R) G_L^K(\tau_L) (A_R A_L - 1) i\epsilon_R \tau_R + i\epsilon_L \tau_L, \quad (71)$$

$$A_R = \prod_{n=\max(0, \tilde{m})}^{\infty} \frac{\Delta_R [\alpha_{R, n-\tilde{m}} w_{\tau_R}(x) + \alpha_{L, n} w_{\tau_L}(x+y)]}{\Delta_R [\alpha_{R, n-\tilde{m}} w_{\tau_R}(x)] \Delta_R [\alpha_{L, n} w_{\tau_L}(x)]}, \quad (72)$$

$$A_L = \prod_{n=\max(0, \tilde{m})}^{\infty} \frac{\Delta_L [\alpha_{R, n-\tilde{m}} w_{\tau_L}(x-y) + \alpha_{L, n} w_{\tau_R}(x)]}{\Delta_L [\alpha_{R, n-\tilde{m}} w_{\tau_L}(x)] \Delta_L [\alpha_{L, n} w_{\tau_R}(x)]}. \quad (73)$$

Here $\tilde{m} = (m + 1)/2$, and we use the notations introduced in Eqs. (63, 64).

Similarly to (66, 67), one can define the irreducible correlation function of the occupation numbers of the right-movers:

$$\langle\langle n_R(x_R, \epsilon_R) n_R(x'_R, \epsilon'_R) \rangle\rangle = \frac{V_F^2}{4} \int d\tau_R d\tau'_R M_{RR}(x_R, \tau_R; x'_R, \tau'_R) e^{i\epsilon_R \tau_R + i\epsilon'_R \tau'_R}, \quad (74)$$

$$M_{RR}(x_R, \tau_R; x'_R, \tau'_R) = \langle [\psi_R(x_R, \frac{\tau_R}{2}), \psi_R^+(x_R, -\frac{\tau_R}{2})] [\psi_R(x'_R, \frac{\tau'_R}{2}), \psi_R^+(x'_R, -\frac{\tau'_R}{2})] \rangle + G_R^K(\tau_R) G_R^K(\tau'_R). \quad (75)$$

In order to be able to interpret (74) as the correlator of the distribution functions, we assume below that the two pairs of ψ -operators in (75) are inserted far from each other, so that $X_{RR} \equiv x_R - x'_R \gg l_{T^*}$. Under this assumption, just as in the case of R - L correlations, (75) is the only one non-trivial correlation function of four right electronic fields. Consideration of phases $\delta_\eta(x)$ corresponding to (75) shows that the (74) is non-zero provided that $X_{RR} \approx 2Klm$ and can be decomposed as (we set $\tilde{m} = m/2$):

$$\langle\langle n_R(x_R, \epsilon_R) n_R(x'_R, \epsilon'_R) \rangle\rangle = \sum_{m \in \text{even}, m \neq 0} f_m^{RR}(\epsilon_R, \epsilon_L, X_{RR} - mKl), \quad (76)$$

$$f_m^{RR}(\epsilon_R, \epsilon_L, y) = -\frac{V_F^2}{4} \int d\tau_R d\tau'_R G_R^K(\tau_R) G_L^K(\tau_L) (A_R A_L - 1) e^{i\epsilon_R \tau_R + i\epsilon'_R \tau'_R}, \quad (77)$$

$$A_R = \prod_{n=\max(0, \tilde{m})}^{\infty} \frac{\Delta_R [\alpha_{R, n-\tilde{m}} w_{\tau_R}(x) + \alpha_{R, n} w_{\tau'_R}(x+y)]}{\Delta_R [\alpha_{R, n-\tilde{m}} w_{\tau_R}(x)] \Delta_R [\alpha_{R, n} w_{\tau'_R}(x)]}, \quad (78)$$

$$A_L = \prod_{n=\max(0, \tilde{m})}^{\infty} \frac{\Delta_L [\alpha_{L, n-\tilde{m}} w_{\tau_R}(x) + \alpha_{L, n} w_{\tau'_R}(x-y)]}{\Delta_L [\alpha_{L, n-\tilde{m}} w_{\tau_R}(x)] \Delta_L [\alpha_{L, n} w_{\tau'_R}(x)]}. \quad (79)$$

The term with $m = 0$ is omitted from the summation due to our assumption that $X_{RR} > l_{T^*}$. Again, investigation of the functions $f_m^{RR}(\epsilon_R, \epsilon_L, x)$ requires evaluation of the functional determinants Δ_η .

The physics of obtained correlations is discussed in Sec. V C.

C. Discussion

We assert that the correlations discovered in Sec. VB represent a quantum interference effect. To clarify this point, we make a digression and consider a very simple case of one right boson with momentum q populating the wire. This situation is described by the density matrix

$$\hat{\rho}_{1b} = b_{R,q}^{+in}|0\rangle\langle 0|b_{R,q}^{in}, \quad (80)$$

where $|0\rangle$ is the ground state. Using the connection of the in- and out-bosons and then performing the refermionization we translate ρ_{1b} into the out-fermions

$$\hat{\rho}_{1b} = \frac{2\pi}{Lq} \sum_{k,k'} \left(t_q a_{R,k+q}^{+out} a_{R,k}^{out} + r_{R,q} a_{L,k-q}^{+out} a_{L,k}^{out} \right) |0\rangle\langle 0| \left(t_q^* a_{R,k'-q}^{+out} a_{R,k}^{out} + r_{R,q}^* a_{L,k'+q}^{+out} a_{L,k}^{out} \right). \quad (81)$$

We can now evaluate the correlation function of the left and right distributions directly in the fermionic language and get

$$\begin{aligned} \langle \langle n_R(x_R, \epsilon_R) n_L(x_L, \epsilon_L) \rangle \rangle &= -\delta n_R(\epsilon_R) \delta n_L(\epsilon_L) \\ &+ \frac{2\pi}{Lq} (n_{R+}^0 - n_{R-}^0) (n_{L+}^0 - n_{L-}^0) \left(r_{R,q}^* t_q e^{iq(x_R+x_L)} + r_{R,q} t_q^* e^{-iqX_{RL}} \right). \end{aligned} \quad (82)$$

Here $\omega = qV_F$ is the frequency of the boson in the system; $n_{\eta\pm}^0 = \Theta(-(\epsilon_\eta \pm \omega/2))$ and

$$\delta n_R(\epsilon_R) \equiv n_R^{out}(\epsilon_R) - n^0(\epsilon_R) = \frac{2\pi}{Lq} |t_q|^2 (2n^0(\epsilon_R) - n^0(\epsilon_R + \omega) - n^0(\epsilon_R - \omega)), \quad (83)$$

$$\delta n_L(\epsilon_L) \equiv n_L^{out}(\epsilon_L) - n^0(\epsilon_L) = \frac{2\pi}{Lq} |r_{R,q}|^2 (2n^0(\epsilon_L) - n^0(\epsilon_L + \omega) - n^0(\epsilon_L - \omega)). \quad (84)$$

$$(85)$$

The two terms in (82) have distinct physical origin. The first one originates from the probabilistic nature of the boson transmission-reflection process. It would remain unchanged upon the replacement of the density matrix by the statistical mixture

$$\tilde{\rho}_{1b} = |t_q|^2 b_{R,q}^{+out}|0\rangle\langle 0|b_{R,q}^{out} + |r_{R,q}|^2 b_{L,-q}^{+out}|0\rangle\langle 0|b_{L,-q}^{out}. \quad (86)$$

This term favors *anti-correlations* of electrons at energies of equal signs. This is a typical anti-correlation between the mutually exclusive events. Indeed, suppose the boson we have injected into the system was transmitted through the interacting part of the wire. In this case at the output of our device there is a particle-hole excitation in the right branch, i.e. a right electron at positive energy and a right hole at negative energy. At the same time the left branch has no excitation. On the contrary, if the boson was reflected there is a left electron at positive energy and a left hole. The right branch is empty. Thus, in such a probabilistic description there is no way to have an excited electron both in the left and right branches, which results in anti-correlations between the fermionic occupation numbers at energies of the same sign.

Note that the probabilistic contribution to the correlator does not depend on the coordinates where the occupation numbers are measured. This is quite general. Suppose we have some complicated density matrix $\hat{\rho}$ in terms of in-bosons. We translate it into the out-bosons. It is easy to see that if we now switch to the classical description of the system by dropping all the non-diagonal elements of the density matrix (cf. transition from (81) to (86)) we will end up with the correlator of the occupation numbers independent on X_{RL} . Only the matrix elements of $\hat{\rho}$ involving momentum transfer from left to right branch can provide such a dependence.

The second contribution to the correlator (82) is due to the fact that on the quantum level the scattering of a boson creates *coherent* superposition of the state with a boson in the right branch and a state with a boson in the left branch. Thus, the excited particle-hole pair is (virtually) simultaneously present in *both* branches. To elucidate the effect of this quantum term, let us consider a slightly more general density matrix which is a statistical mixture of (80):

$$\hat{\rho}_{mix} = \sum_q c_q b_{R,q}^{+in}|0\rangle\langle 0|b_{R,q}^{in}, \quad \sum_q c_q = 1. \quad (87)$$

Now the quantum contribution is modified accordingly,

$$\langle\langle n_R(x_R, \epsilon_R) n_L(x_L, \epsilon_L) \rangle\rangle_{\text{quantum}} = -4\mathcal{T}^2 \mathcal{R} \sum_q c_q \frac{2\pi}{Lq} (n_{R+}^0 - n_{R-}^0) (n_{L+}^0 - n_{L-}^0) \frac{\sin Kql \sin qX_{RL}}{|1 - \mathcal{R}^2 e^{2iKql}|^2}. \quad (88)$$

We have used here the explicit expressions (22) for the transmission and reflection amplitudes in the sharp-boundary model. Let the coefficients c_q be peaked at some $q = q_0$. Under the assumption that the peak width is much larger than $1/Kl$ and the energies $\epsilon_{R(L)}$ are not too close to $q_0 V_F/2$, we can average the expression under the sum over fast oscillations on the scale $1/Kl$. The result is non-vanishing only if X_{RL} is close to odd multiple of Kl , $X_{RL} = mKl + x$, $m \in \text{odd}$, in which case

$$\langle\langle n_R(x_R, \epsilon_R) n_L(x_L, \epsilon_L) \rangle\rangle_{\text{quantum}} = -\frac{4\pi\mathcal{T}^2 \mathcal{R}^m \text{sign } m}{1 + \mathcal{R}^2} \sum_q c_q \frac{2\pi}{Lq} (n_{R+}^0 - n_{R-}^0) (n_{L+}^0 - n_{L-}^0) \cos qx. \quad (89)$$

This result is in agreement with the general coordinate dependence (70) of the correlator of fermionic distributions.

The density matrices $\hat{\rho}_{1b}$ and $\hat{\rho}_{mix}$ are very simple as they contain just one fermionic excitation. The truly non-equilibrium density matrix (41), when written in terms of bosons, is much more complicated. It contains infinitely many terms representing multiple-boson processes. To collect properly their contributions into the correlation function of left and right occupation numbers is the task accomplished by the functional determinants $\Delta_\eta[\delta_\eta(x)]$. In general, both classical noise and quantum interference contribute to the result. The lesson we learnt from the analysis of simple density matrices allows us to identify clear manifestations of quantum effects. First, this is the dependence of the correlation functions on the coordinates, see Eq. (70). Second, these are positive correlations between distribution functions of left and right movers at energies of the same sign. We will see in Sec. VD below that the correlation functions of a non-equilibrium LL do show both these features.

D. Partial equilibrium

A detailed analysis of the functions f_m^{RL} describing the correlations in electronic distributions requires a careful investigation of the determinants $\Delta_\eta[\delta_\eta(x)]$ with given distributions of the incoming electrons. In general, these determinants can not be evaluated analytically, except for asymptotic long-time behavior³³ (which is not sufficient for our purposes here). Thus, one has to resort to numerics in order to achieve the comprehensive understanding of the correlations in the fermionic occupation numbers.

Relegating such a numerical analysis for a future work, we focus below on the case of partial equilibrium that can be treated analytically. This is the situation when distribution functions of the incoming fermions are of the Fermi-Dirac form but with different temperatures T_R and T_L of the left- and right-movers⁵². The functional determinants $\Delta_\eta[\delta_\eta(x)]$ become under such circumstance exponentially quadratic functionals of phases³¹,

$$\ln \Delta_\eta = -\frac{1}{4\pi} \int_0^\infty \frac{dq}{2\pi} q (B_\eta(qV_F) - 1) |\delta_\eta(q)|^2, \quad (90)$$

where $B_\eta(\omega) = \coth \omega/2T_\eta$ are the bosonic distribution functions.

One can now use (90) to evaluate the products of the determinants (72,73,78,79) analytically. Alternatively, we can apply Eq. (90) to the full functional determinants governing the correlations in the electronic distributions. The Fourier components of the phases δ_η are easily read off from (59). For example, in the case of the correlator of left and right distribution functions we have

$$|\delta_R(q)|^2 = \frac{(4\pi)^2}{q^2} \left[|t_q|^2 \sin^2 \frac{qV_F\tau_R}{2} + |r_q|^2 \sin^2 \frac{qV_F\tau_L}{2} + 2it_q r_q^* \sin \frac{qV_F\tau_R}{2} \sin \frac{qV_F\tau_L}{2} \sin q(x_R + x_L) \right], \quad (91)$$

$$|\delta_L(q)|^2 = \frac{(4\pi)^2}{q^2} \left[|t_q|^2 \sin^2 \frac{qV_F\tau_R}{2} + |r_q|^2 \sin^2 \frac{qV_F\tau_L}{2} - 2it_q r_q^* \sin \frac{qV_F\tau_R}{2} \sin \frac{qV_F\tau_L}{2} \sin q(x_R + x_L) \right]. \quad (92)$$

Plugging this into (90), averaging the expression under the integral over fast oscillations of the bosonic transmission and reflection amplitudes (which is equivalent to the neglect of the interference of non-overlapping pulses in $\delta_\eta(x)$ discussed in the previous section) and using the standard equality

$$-\int_0^{+\infty} \frac{dx}{x} (1 - \cos \alpha x) \left(\coth \frac{x}{2} - 1 \right) = \ln \frac{\pi \alpha}{\text{sh } \pi \alpha}, \quad (93)$$

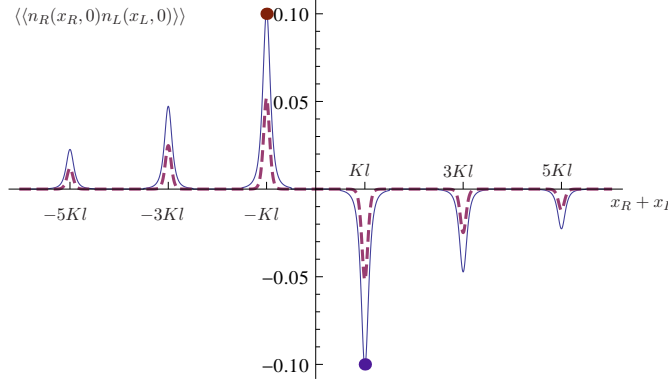


FIG. 4: (Color online). The dependence of the correlator $\langle\langle n_R(x_R, 0)n_L(x_L, 0) \rangle\rangle$ on the sum of coordinates $x_R + x_L$. Solid line corresponds to $T_R/T_L = 10$ while the dashed line is the result for $T_R/T_L = 2$. Energy dependence of the correlator of left and right distribution functions at $x_R + x_L = Kl$ (blue dot) and $x_R + x_L = -Kl$ (brown dot) is shown on Fig. 5.

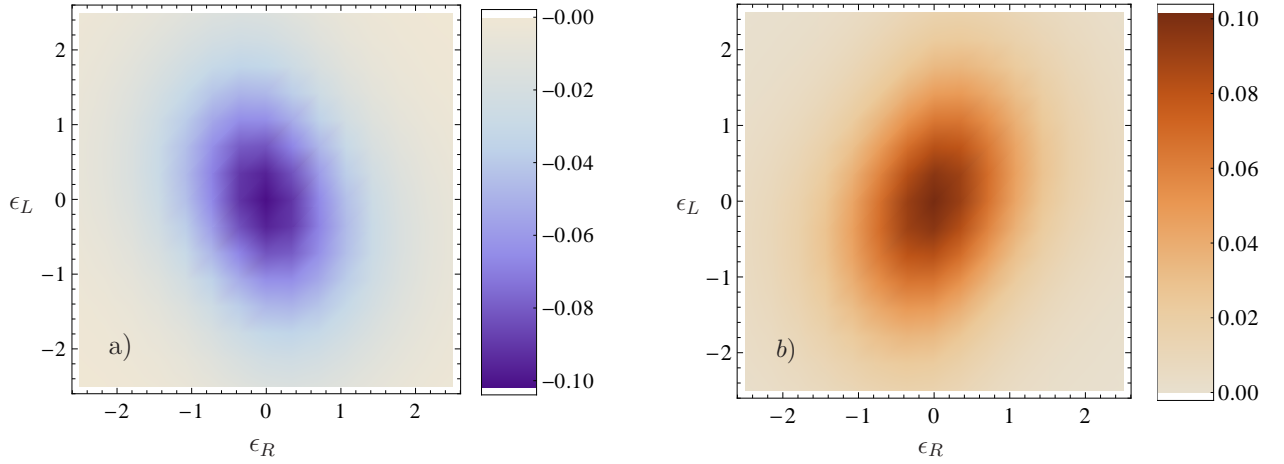


FIG. 5: (Color online). Energy dependence of functions $f_1^{RL}(\epsilon_R, \epsilon_L, 0)$ and $f_{-1}^{RL}(\epsilon_R, \epsilon_L, 0)$ (plots (a) and (b) respectively). All energies are measured in units of the largest temperature $T_R = 10T_L$.

one finds that the correlation function of left and right occupation numbers has indeed the form (70) with

$$f_m^{RL}(\epsilon_R, \epsilon_L, x) = -\frac{V_F^2}{4} \int d\tau_R d\tau_L e^{i\epsilon_L \tau_L + i\epsilon_R \tau_R} G_R^K(\tau_R) G_L^K(\tau_L) \left[\left(\frac{1 - \frac{\text{ch } \pi T_R (\tau_R + \tau_L)}{\text{ch } 2\pi T_R x}}{1 - \frac{\text{ch } \pi T_L (\tau_R + \tau_L)}{\text{ch } 2\pi T_L x}} \frac{1 - \frac{\text{ch } \pi T_L (\tau_R - \tau_L)}{\text{ch } 2\pi T_L x}}{1 - \frac{\text{ch } \pi T_R (\tau_R - \tau_L)}{\text{ch } 2\pi T_R x}} \right)^{\gamma_m} - 1 \right]. \quad (94)$$

Here the exponents γ_m are given by $\gamma_m = \text{sign } m \mathcal{T}^2 \mathcal{R}^{|m|} / (1 + \mathcal{R}^2)$, and the Keldysh Green functions of the outgoing fermions are³¹

$$G_R^K(\tau_R) = \frac{1}{\pi V_F \tau_R} \left[\frac{\pi T_R \tau_R}{\text{sh } \pi T_R \tau_R} \right]^{\frac{\mathcal{T}^2}{1+\mathcal{R}^2}} \left[\frac{\pi T_L \tau_R}{\text{sh } \pi T_L \tau_R} \right]^{\frac{2\mathcal{R}^2}{1+\mathcal{R}^2}}, \quad (95)$$

$$G_L^K(\tau_L) = \frac{1}{\pi V_F \tau_L} \left[\frac{\pi T_L \tau_L}{\text{sh } \pi T_L \tau_L} \right]^{\frac{\mathcal{T}^2}{1+\mathcal{R}^2}} \left[\frac{\pi T_R \tau_L}{\text{sh } \pi T_R \tau_L} \right]^{\frac{2\mathcal{R}^2}{1+\mathcal{R}^2}}. \quad (96)$$

$$(97)$$

The Fourier transformation in (94) can be evaluated numerically. The results are exemplified on Fig. 4, 5 and 6. To produce the graphs we have chosen the LL parameter $K = 0.2$ such that the transmission and reflection amplitudes \mathcal{T} , \mathcal{R} are approximately equal. Figure 4 shows the zero energy correlator $\langle\langle n_R(x_R, 0)n_L(x_L, 0) \rangle\rangle$ in its dependence on $x_R + x_L$. Solid and dashed lines correspond to strong ($T_R/T_L = 10$) and comparatively weak ($T_R/T_L = 2$) non-equilibrium. One observes the characteristic peaks at $x_R + x_L = (2m + 1)Kl$. Occupation numbers of left and right

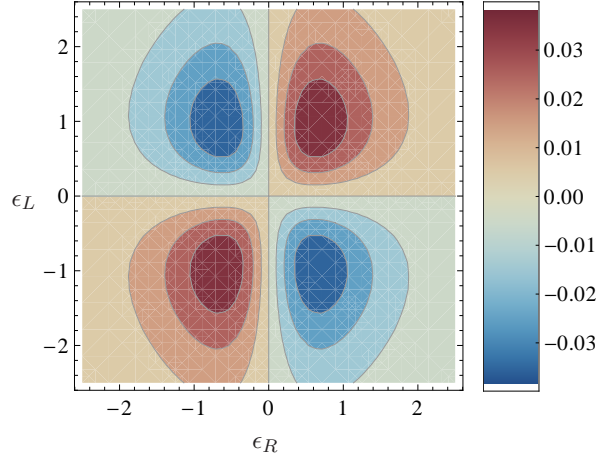


FIG. 6: (Color online). Correlator of the distribution functions of left and right electrons integrated over the $x_R + x_L$ (see Eq. (98)). The ratio of temperatures was taken to be $T_R/T_L = 10$.

movers are anti-correlated for $x_R + x_L > 0$ and correlated at $x_R + x_L < 0$ (of course, the situation will be reversed if one assumes that $T_R < T_L$).

Figure (5) demonstrates the functions $f_{\pm 1}^{RL}(\epsilon_R, \epsilon_L, 0)$. The correlations reach maximum near $\epsilon_R = \epsilon_L = 0$. In the case of f_1^{RL} the anti-correlations are somewhat more extended in the direction $\epsilon_R = -\epsilon_L$ while the correlations in f_{-1}^{RL} prefer to develop for ϵ_R and ϵ_L of equal sign. This fact leads to a remarkable structure in the “mean correlations” of the distribution functions of left- and right- movers as given by

$$h(\epsilon_R, \epsilon_L) \equiv \frac{T_R}{V_F} \int d(x_R + x_L) \langle \langle n_R(x_R, \epsilon_R) n_L(x_L, \epsilon_L) \rangle \rangle. \quad (98)$$

(The prefactor was introduced to make h dimensionless.) The function $h(\epsilon_R, \epsilon_L)$ is shown on Fig. 6. We observe that on average the left- and right- movers are correlated when their energies have equal signs and anti-correlated in the opposite case. Note that this result is opposite to the one that would be except on the basis of the classical consideration of the bosonic transmission-reflection process, see Sec. V C.

A similar analysis can be performed for the case of the correlations in the occupation numbers of right electrons along. One finds the correlation function of right distributions in the form of (76) with

$$f_m^{RR}(\epsilon_R, \epsilon'_R, x) = -\frac{V_F^2}{4} \int d\tau_R d\tau'_R e^{i\epsilon_L \tau_L + i\epsilon_R \tau_R} G_R^K(\tau_R) G_R^K(\tau'_R) \left[\left(\frac{1 - \frac{\text{ch } \pi T_L (\tau_R + \tau'_R)}{\text{ch } 2\pi T_L x}}{1 - \frac{\text{ch } \pi T_R (\tau_R + \tau'_R)}{\text{ch } 2\pi T_R x}} \frac{1 - \frac{\text{ch } \pi T_R (\tau_R - \tau'_R)}{\text{ch } 2\pi T_R x}}{1 - \frac{\text{ch } \pi T_L (\tau_R - \tau'_R)}{\text{ch } 2\pi T_L x}} \right)^{\gamma_m} - 1 \right]. \quad (99)$$

The exponents γ_m are given by

$$\gamma_m = \mathcal{T}^2 \mathcal{R}^{|m|} / (1 + \mathcal{R}^2). \quad (100)$$

The analogous correlator of the left distributions can be obtained via simple exchange $T_R \leftrightarrow T_L$. Provided that $T_R > T_L$ all the functions f_m^{RR} turn out to be positive, while the corresponding functions f_m^{LL} are negative (cf. Fig. 7 showing $f_2^{RR}(\epsilon_R, \epsilon'_R, 0)$ and $f_2^{LL}(\epsilon_L, \epsilon'_L, 0)$). Thus, the hotter electrons have positive correlations built into their distribution, while the colder ones are anti-correlated.

VI. CONCLUSIONS

In this paper we have developed an operator approach to the non-equilibrium LL. Using bosonization and refermionization techniques, we have explicitly determined the many-body density matrix of the system and derived the fermionic correlation functions in terms of Fredholm determinants Δ_η . Let us note that usually the correlation functions of the many-body interacting system can only be found approximately by truncating the Bogoliubov-Born-Green-Kirkwood-Yvon chain. The model considered in this work constitutes a remarkable example of a many-body problem where all the correlation functions can be evaluated exactly.

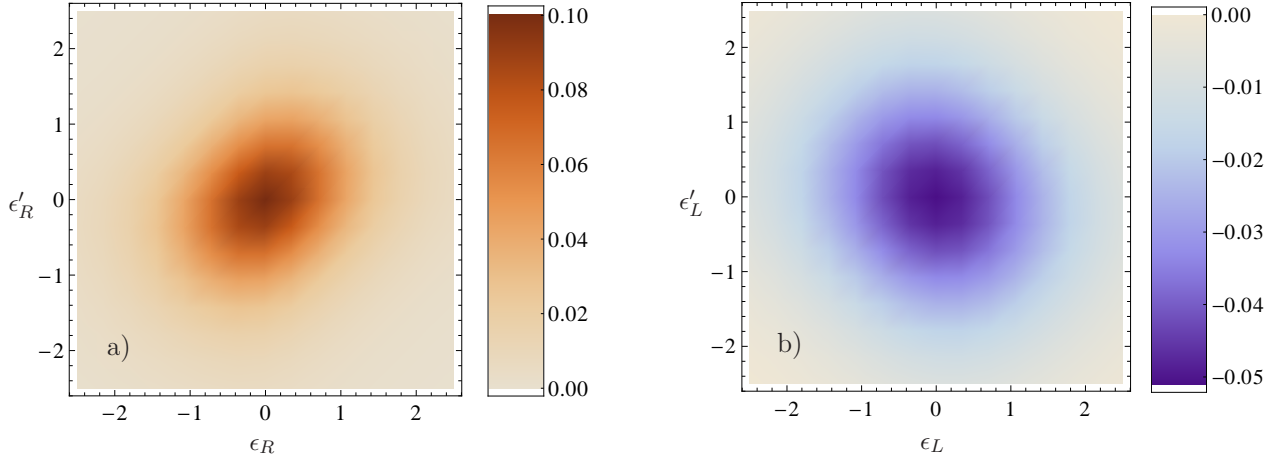


FIG. 7: (Color online). The energy dependence of the correlators $\langle\langle n_R(\epsilon_R, x_R) n_R(\epsilon'_R, x'_R) \rangle\rangle$ and $\langle\langle n_L(\epsilon_L, x_L) n_L(\epsilon'_L, x'_L) \rangle\rangle$ for $x_\eta - x'_\eta = 2Kl$. While the occupation numbers of the hotter (right in this case) electrons are correlated, the occupation numbers of colder electrons are anti-correlated.

We have employed our technique to study the four-point correlation functions of the electrons coming out of the LL wire. While the *dynamics* of the outgoing electrons is free, the corresponding density matrix is highly complicated. It incorporates the correlations $\langle\langle n_\eta(x_\eta, \epsilon_\eta) n_{\eta'}(x_{\eta'}, \epsilon_{\eta'}) \rangle\rangle$ in the electronic distribution functions caused by the scattering of the LL bosons at the boundaries between the LL wire and the non-interacting leads. The spatial dependence of these correlations can be deduced from the general analysis of the corresponding functional determinants Δ_η . It displays characteristic interference picks (or dips) of the width l_{T^*} at distance Kl (the “optical length” of the interacting wire) one from another, see Fig. 4.

For the case of partial equilibrium we have evaluated the Fredholm determinants governing the correlation function $\langle\langle n_\eta(x_\eta, \epsilon_\eta) n_{\eta'}(x_{\eta'}, \epsilon_{\eta'}) \rangle\rangle$ analytically. We have found a non-trivial spatial dependence of correlations, both in the occupation numbers of electrons of the same chirality and between the occupation numbers of left- and right-movers. Within the “hotter” chiral branch, the positive correlation are developed in the electronic distribution. On the other hand, anti-correlations are seen in the distribution of colder electrons (see Fig. 7). The sign of the correlation function $\langle\langle n_R(x_R, \epsilon_R) n_L(x_L, \epsilon_L) \rangle\rangle$ depends on the sign of $x_R + x_L$. On average, the occupation numbers of right- and left-movers are correlated at energies of the same sign and anti-correlated at energies of opposite signs, see Fig. 6. The obtained results indicate that quantum-interference effects contribute crucially to the correlation functions. Intrinsically quantum correlations between parts of the system which can not be understood in a classical framework are referred to as entanglement⁵³. In recent years it was recognized that quantifying the entanglement between subsystems of a many-body quantum system can provide a clue to many relevant properties of the system, see e.g. Ref. 54,55. Such quantification for a *mixed* state of a quantum system is in general a highly complicated task⁵⁶. It remains to be seen to what extent the correlations found in this work are relevant in the quantum information context.

We stress once again that the correlations studied are a genuine non-equilibrium effect absent in equilibrium LL. These correlations are an experimentally relevant quantity and can be measured in a specifically designed tunneling experiment. For example, to access the correlator of left and right distribution functions, one can imagine the following setup. Suppose that two tunneling probes are attached to the wire at points x_R, x_L satisfying $\eta x_\eta > l/2$. We can also assume for simplicity that the right probe allows only the tunneling of right electrons, while only left electrons can tunnel through the left one. Then (under the assumption that the tunneling densities states in the probes are not flat) the correlator of right and left tunneling currents $\langle\langle I_R(V_R) I_L(V_L) \rangle\rangle$ should be sensitive to the correlations in the distribution functions. The voltages V_η biasing the probes will control the corresponding energies in $\langle\langle n_R(x_R, \epsilon_R) n_L(x_L, \epsilon_L) \rangle\rangle$. Clearly, a more sophisticated analysis is needed to extract the information on the correlator of the distribution functions from the correlation in the tunneling currents in a realistic setup. Our analysis shows, however, that the correlator of the distribution functions is in principle a measurable quantity.

Concluding the paper, we briefly discuss possible extensions of the present work. First, under the general non-equilibrium conditions, a numerical analysis of the functional determinants Δ_η is needed to fully understand the correlations in the electronic occupation numbers. Second, the formalism developed here can be extended to cover some non-stationary states of the LL. For example, in view of the recent experimental developments⁵⁷, it is very interesting to investigate a LL wire exposed to on-demand coherent single-electron sources. In particular, in the context quantum information processing, especially intriguing is the entanglement generated by interaction between two electrons injected by single-electron sources from the left and right leads. Third, upon a proper modification, our

technique should also be useful for the investigation of the fractional quantum Hall edge states out of equilibrium. In this systems it is very interesting to look for manifestation of the fractionally-charged quasiparticles in the non-equilibrium correlation functions.

VII. ACKNOWLEDGEMENTS

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Appendix A: Density matrix of non-equilibrium fermions in the bosonic representation

In this appendix, we transform the density matrix (41) into bosonic representation. Throughout this Appendix we will be dealing with the in-fermions a^{in} and in-bosons b^{in} only. In order to simplify notations, we omit the index “in” for all the operators. Further, we focus on right-moving in-fermions. The contribution of right-movers is obtained analogously; the total density matrix is the product of contributions of left and right in-particles.

We are thus looking for the bosonic representation of the statistical operator

$$\hat{\rho} = \frac{1}{Z} \exp \left[- \sum_k \epsilon(k) (a_k^\dagger a_k - n^0(k)) \right]. \quad (\text{A1})$$

Here Z is the normalization factor, $n^0(k) = \Theta(-k)$ is the ground-state distribution function and $\epsilon(k)$ determines the distribution of the electrons via

$$n(k) = \frac{1}{1 + e^{\epsilon(k)}}. \quad (\text{A2})$$

A particularly convenient basis in the Hilbert space of chiral fermions is provided by the bosonic coherent states which are the eigenstates of the bosonic annihilation operators. Each coherent state is labeled by the total number of fermions N and the set of eigenvalues β_q of the operators b_q . Explicitly

$$|N, \beta\rangle = \exp \left[\sum_{q>0} \beta_q b_q^\dagger \right] |N\rangle. \quad (\text{A3})$$

The coherent states (A3) form the overcomplete basis with the resolution of identity given by

$$1 = \sum_N \int \left[\prod_{q>0} d\beta_q^* d\beta_q \right] \exp \left[- \sum_{q>0} |\beta_q|^2 \right] |N, \beta\rangle \langle N, \beta^*|. \quad (\text{A4})$$

The overlap of two coherent states is given by

$$\langle M, \beta^* | N, \beta \rangle = \delta_{M,N} \exp \left[\sum_{q>0} |\beta_q|^2 \right]. \quad (\text{A5})$$

We are interested in the matrix elements of the statistical operator $\hat{\rho}$ in the basis of the coherent states. They are given by (obviously, $\hat{\rho}$ is diagonal with respect to the total number of fermions)

$$\langle \beta^*, N | \hat{\rho} | N, \beta \rangle = \langle N | \exp \left[\sum_{q>0} \beta_q^* b_q \right] \hat{\rho} \exp \left[\sum_{q>0} \beta_q b_q^\dagger \right] | N \rangle. \quad (\text{A6})$$

We remind that the bosons are proportional to the Fourier components of the fermionic density:

$$b_q^\dagger = \sqrt{\frac{2\pi}{L|q|}} \sum_k a_{k+q}^\dagger a_k, \quad b_q = \sqrt{\frac{2\pi}{L|q|}} \sum_k a_{k-q}^\dagger a_k. \quad (\text{A7})$$

Thus, the operator under the average in (A6) is exponentially quadratic in fermions.

It is convenient to introduce new fermions:

$$c_k = a_k (1 - n^N(k)) + a_k^+ n^N(k), \quad (\text{A8})$$

$$c_k^+ = a_k^+ (1 - n^N(k)) + a_k n^N(k). \quad (\text{A9})$$

Here $n^N(k) = \Theta(-k + \frac{2\pi}{L}N)$ is the distribution function of a Fermi sea with N extra particles. The idea behind the transformation (A9) is that the state $|N\rangle$ is nullified by all the operators c_k , which simplifies the derivation. In terms of the new fermions c we have

$$\sum_{q>0} \beta_q b_q^+ = \sum_{k_1, k_2} c_{k_1}^+ c_{k_2} U_{k_1, k_2} + \sum_{k_1, k_2} c_{k_1}^+ c_{k_2}^+ V_{k_1, k_2}. \quad (\text{A10})$$

The matrices U_{k_1, k_2} and V_{k_1, k_2} carrying two momentum indices are given by

$$U = \Phi(1 - n^N) - \Phi^T n^N, \quad (\text{A11})$$

$$V = \frac{1}{2} [\Phi + \Phi^T, n^N]. \quad (\text{A12})$$

Here the distribution function n^N is considered as a matrix diagonal in momentum space, while the matrix Φ has the matrix elements

$$\Phi_{k_1 - k_2} = \Theta(k_1 - k_2) \sqrt{\frac{2\pi}{L(k_1 - k_2)}} \beta_{k_1 - k_2}. \quad (\text{A13})$$

Using the fermionic commutation relations satisfied by c_k and the fact that all c_k annihilate $|N\rangle$, one now finds

$$\exp \left[\sum_{q>0} \beta_q b_q^+ \right] |N\rangle = \exp \left[\sum_{k_1 k_2} c_{k_1}^+ c_{k_2}^+ W_{k_1 k_2} \right] |N\rangle, \quad (\text{A14})$$

$$W = \int_0^1 ds \exp[sU] V \exp[sU^T]. \quad (\text{A15})$$

Note that the operator Φ preserves the subspace $k > \frac{2\pi}{L}N$ while the operator Φ^T preserves the subspace $k \leq \frac{2\pi}{L}N$. On the other hand, on the first of these subspaces $1 - n^N = 1$, while the other subspace is nullified by $1 - n^N$. It follows that

$$e^{sU} = e^{s\Phi}(1 - n^N) + e^{-s\Phi^T} n^N. \quad (\text{A16})$$

Using this relation, the expression for the matrix W can be simplified to the form

$$W = \frac{1}{2} \left[e^{-\Phi^T} (1 - n^N) e^{\Phi^T} - e^{\Phi} (1 - n^N) e^{-\Phi} \right]. \quad (\text{A17})$$

The matrix element (A6) now reads

$$\begin{aligned} \langle \beta^*, N | \hat{\rho} | N, \beta \rangle &= \frac{\gamma}{Z} \langle N | \exp \left[\sum_{k_1 k_2} c_{k_1} c_{k_2} W_{k_1 k_2}^+ \right] \exp \left[- \sum_{k_1 k_2} c_{k_1}^+ c_{k_2} E_{k_1 k_2} \right] \exp \left[\sum_{k_1 k_2} c_{k_1}^+ c_{k_2}^+ W_{k_1 k_2} \right] | N \rangle \\ &\equiv \frac{\gamma}{Z} \langle N | e^{\widehat{W}^+} e^{\widehat{E}} e^{\widehat{W}} | N \rangle, \end{aligned} \quad (\text{A18})$$

where

$$E_{k_1, k_2} = \epsilon(k_1) (1 - 2n^N(k_1)) \delta_{k_1 k_2}, \quad \gamma = \exp \left[- \sum_k \epsilon(k) (n^N(k) - n^0(k)) \right]. \quad (\text{A19})$$

For the evaluation of the average (A18) one can employ the standard technique of the fermionic coherent states. Let us define the set of the eigenstates of annihilation operators c_k with the Grassmann eigenvalues χ_k (χ stands for the full set of χ_k with $k = -\infty \dots \infty$):

$$c_k |\chi\rangle = \chi_k |\chi\rangle. \quad (\text{A20})$$

The coherent states $|\chi\rangle$ form an overcomplete basis with the resolution of identity

$$1 = \int D\chi^* D\chi \exp \left[- \sum_k \chi_k^* \chi_k \right] |\chi\rangle \langle \chi^*|, \quad (\text{A21})$$

where

$$D\chi^* D\chi = \prod_k d\chi_k^* d\chi_k. \quad (\text{A22})$$

Using the resolution of identity (A21) and noting that the state $|N\rangle$ is itself a coherent state $|\chi = 0\rangle$, we get

$$\begin{aligned} \langle \beta^*, N | \hat{\rho} | N, \beta \rangle &= \frac{\gamma}{Z} \int D\chi^{1*} D\chi^1 D\chi^{2*} D\chi^2 \exp \left[- \sum_k (\chi_k^{1*} \chi_k^1 + \chi_k^{2*} \chi_k^2) \right] \\ &\quad \times \langle \chi^* = 0 | e^{\widehat{W}^+} |\chi^1\rangle \langle \chi^{1*} | e^{\widehat{E}} |\chi^2\rangle \langle \chi^{2*} | e^{\widehat{W}} | \chi = 0 \rangle. \end{aligned} \quad (\text{A23})$$

Finally, since

$$\langle \chi^* | e^{\widehat{W}} | \chi \rangle = \exp \left[\sum_k \chi_k^* \chi_k + \sum_{k_1, k_2} \chi_{k_1}^* \chi_{k_2}^* W_{k_1, k_2} \right], \quad (\text{A24})$$

$$\langle \chi^* | e^{\widehat{E}} | \chi \rangle = \exp \left[\sum_{k_1, k_2} \chi_{k_1}^* \chi_{k_2}^* [e^{-E}]_{k_1 k_2} \right], \quad (\text{A25})$$

$$(\text{A26})$$

we are left with a simple Gaussian integral over Grassman variables. The result of the integration reads

$$\langle \beta^*, N | \hat{\rho} | N, \beta \rangle = \frac{\gamma}{Z} \left[\det \begin{pmatrix} 1 & -2e^{-E} W e^{-E} \\ 2W^+ & 1 \end{pmatrix} \right]^{1/2}. \quad (\text{A27})$$

Using the explicit form of the matrices W and E , one can reduce the expression above to

$$\langle \beta^*, N | \hat{\rho} | N, \beta \rangle = \frac{\gamma}{Z} \det \left[e^{-\epsilon} e^{\Phi} (1 - n_N) e^{-\Phi} e^{\epsilon} + e^{-\Phi^+} n_N e^{\Phi^+} \right]. \quad (\text{A28})$$

Finally, working out the factor in front of the determinant in (A28), we find

$$\langle \beta^*, N | \hat{\rho} | N, \beta \rangle = \det \left[1 - e^{\Phi} n_N e^{-\Phi} (1 - n) - e^{-\Phi^+} (1 - n_N) e^{\Phi^+} n \right]. \quad (\text{A29})$$

Equation (A29) gives an explicit expression for the density matrix of non-equilibrium interacting fermions in the bosonic basis. It has a form of a one-dimensional functional determinant of Fredholm type.

Let us verify that Eq. (A29) reduces to the Boltzmann-Gibbs form at equilibrium, which in present notations corresponds to $\epsilon(k) = kV_F/T$. Indeed, we know that for $\epsilon(k) \equiv 0$ Eq. (A28) should give just the norm of the coherent $|N, \beta\rangle$ (up to the normalization factor Z)

$$\langle \beta^*, N | \hat{\rho} | N, \beta \rangle|_{\epsilon(k)=0} = \frac{\gamma}{Z} \det \left[e^{\Phi} (1 - n_N) e^{-\Phi} + e^{-\Phi^+} n_N e^{\Phi^+} \right] = \frac{1}{Z} \exp \left[\sum_{q>0} |\beta_q|^2 \right]. \quad (\text{A30})$$

On the other hand for $\epsilon(k) = kV_F/T$ we can write

$$\langle \beta^*, N | \hat{\rho}_{eq} | N, \beta \rangle = \frac{\gamma}{Z} \det \left[e^{-\epsilon} e^{\Phi} e^{\epsilon} (1 - n_N) e^{-\epsilon} e^{-\Phi} e^{\epsilon} + e^{-\Phi^+} n_N e^{\Phi^+} \right] = \frac{\gamma}{Z} \det \left[e^{\tilde{\Phi}} (1 - n_N) e^{-\tilde{\Phi}} + e^{-\Phi^+} n_N e^{\Phi^+} \right], \quad (\text{A31})$$

where

$$\tilde{\Phi}_{k_1 - k_2} = [e^{-\epsilon} \Phi e^{\epsilon}]_{k_1, k_2} = \Theta(k_1 - k_2) \sqrt{\frac{2\pi}{L(k_1 - k_2)}} \beta_{k_1 - k_2} e^{-V_F(k_1 - k_2)/T}. \quad (\text{A32})$$

We see that the matrix elements of the equilibrium density matrix can be obtained from the norm of the coherent states just by the replacement

$$\beta_q \rightarrow \beta_q e^{-V_F q/T}, \quad (\text{A33})$$

with the result

$$\langle \beta^*, N | \hat{\rho}_{eq} | N, \beta \rangle = \frac{1}{Z} \exp \left[\sum_{q>0} e^{-qV_F/T} |\beta_q|^2 \right]. \quad (\text{A34})$$

In the operator form, Eq. (A34) reads

$$\hat{\rho}_{eq} = \frac{1}{Z} \exp \left[-\frac{V_F}{T} \sum_{q>0} q b_q^+ b_q \right], \quad (\text{A35})$$

which is the expected equilibrium result.

Away from equilibrium, the bosonic density matrix (A29) is much more complicated. Nevertheless, it can be used to derive the expressions (47, 48, 49) for the prototypical average (43) required for evaluation of correlation functions. To do this, one represents the determinant (A29) as an a Gaussian integral over auxiliary Grassmann variables and performs an expansion in powers of e^Φ . At every order, evaluation of the correlator of the bosonic exponents (43) is then given by a Gaussian integral over bosons and fermions. The crucial point is some specific cancellations between the fermionic and bosonic integrations, which finally allows us to evaluate all orders of the expansion analytically and resum the whole series. This brute-force derivation requires, however, rather cumbersome combinatorics, and we do not present it here. An alternative, considerably simpler, derivation involves re-fermionization, as explained in the main text and Appendix B.

Appendix B: Averaging exponential of bosonic fields in a non-equilibrium Luttinger liquid via re-fermionization

In this Appendix we evaluate the prototypical correlation function (43) where the average is performed with the density matrix (41). Since the right-moving in-fermions are completely independent from the left-moving ones (see Eq. (41)) we ignore the latter for a while and include the left-movers in the final formulas. In our notations we also suppress the index "in" since all the operators (bosons and fermions) we will be dealing with in this Appendix are in-operators and the suppression should not cause any confusion. Thus, we need to evaluate the average

$$Z[g^*, g] = \left\langle \exp \left[\sum_{q>0} g(q) b_q^+ \right] \exp \left[-\sum_{q>0} g^*(q) b_q \right] \right\rangle \quad (\text{B1})$$

with the density matrix

$$\hat{\rho} = \frac{1}{Z} \exp \left[-\sum_k \epsilon(k) (a_k^+ a_k - n^0(k)) \right]. \quad (\text{B2})$$

We can now apply the well-known expression for the trace over the fermionic Hilbert space of a product of exponentially quadratic operators⁵⁸:

$$\text{tr } e^{H_1} \dots e^{H_n} = \det (1 + e^{h_1} \dots e^{h_n}), \quad (\text{B3})$$

where

$$H_i = \sum_{k_1, k_2} h_i^{k_1, k_2} a_{k_1}^+ a_{k_2}. \quad (\text{B4})$$

The right-hand side of (B3) is a determinant of an operator acting in the single-particle Hilbert space. Applying (B3) to the average in question, we get

$$Z[g^*, g] = \det \left(1 - n(k) + e^{-i\delta(x)} n(k) \right), \quad (\text{B5})$$

$$\delta(x) = i \sum_{q>0} \sqrt{\frac{2\pi}{Lq}} (g_q e^{iqx} - g_q^* e^{-iqx}). \quad (\text{B6})$$

The determinant in (B5) is in fact not well defined, for the following reason. Strictly speaking, Eq. (B3) assumes that in the fermionic Hilbert space there is a state nullified by all the operators a_k . This is not the case in the present situation. We can resolve this difficulty by making the transformation from the particle operators a_k , a_k^\dagger to particle and hole operators:

$$c_k = a_k (1 - n^0(k)) + a_k^\dagger n^0(k), \quad (\text{B7})$$

$$c_k^\dagger = a_k^\dagger (1 - n^0(k)) + a_k n^0(k). \quad (\text{B8})$$

Then we can evaluate $Z[g^*, g]$ with the technique of fermionic coherent states. The calculation is very similar to that of Appendix A.

We can make a short-cut, however, if we notice that $Z[g^*, g]$ is an average of an operator normal-ordered in bosons. Thus, $Z[g^*, g] = 1$ at zero temperature. This suggests that the regularized version of Eq. (B5) should be

$$Z[g^*, g] = \det \left[\left(1 - n^0(k) + e^{-i\delta(x)} n^0(k) \right)^{-1} \left(1 - n(k) + e^{-i\delta(x)} n(k) \right) \right] \quad (\text{B9})$$

A direct calculation along the lines of Appendix A indeed confirms this result.

Incorporating now the left electrons into our consideration, we come to Eqs. (47, 48, 49).

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